Cambridge O Level

ADDITIONAL MATHEMATICS   **4037/22**

Paper 2   **May/June 2024**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **12** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1   Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2   Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3   Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4   Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5   Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6   Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.
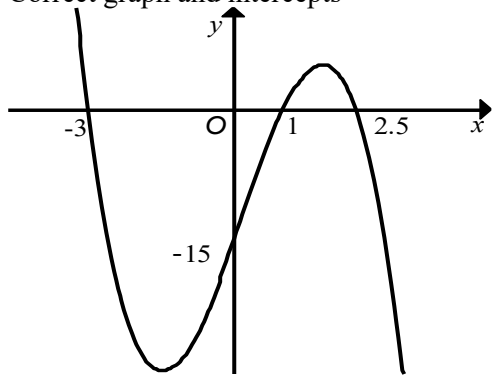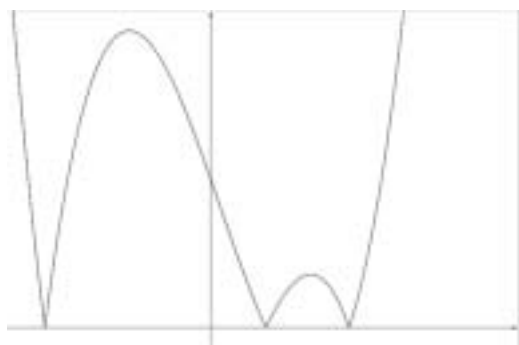
Types of mark

M   Method marks, awarded for a valid method applied to the problem.

A   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B   Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt   answers which round to
cao   correct answer only
dep   dependent
FT   follow through after error
isw   ignore subsequent working
nfww   not from wrong working
oe   or equivalent
rot   rounded or truncated
SC   Special Case
soi   seen or implied

Question	Answer	Marks	Partial Marks
1(a)	Correct graph and intercepts	B3	**B1** for correct shape; the ends must extend above and below the x-axis **B1** for correct roots indicated; must have attempted a cubic shape **B1** for correct y-intercept indicated; must have attempted a cubic shape

Question	Answer	Marks	Partial Marks
1(b)(i)	$-3 \leqslant x \leqslant 1$, $x \geqslant 2.5$ mark final answer	**B2**	**FT** *their* **(a)** providing it is an equivalent cubic shape and has 3 stated or indicated roots for **B2, B1** or **SC1** **B1** for one correct inequality out of two If 0 scored then **SC1** for $-3 < x < 1$, $x > 2.5$ or $-3 < x \leqslant 1$, $x > 2.5$ or $-3 \leqslant x < 1$, $x > 2.5$
1(b)(ii)	Graph of correct shape, with cusps, positive *y*-intercept and *x*-intercepts which match **(a)** 	**B1**	**FT** *their* **(a)** providing it is an equivalent cubic shape
2(a)	$\left[4\sin\dfrac{x}{4} \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$	**B2**	**B1** for $k\sin\dfrac{x}{4}$ where $k>0$ or $k=-4$
	$4\sin\dfrac{\pi}{8} - 4\sin\dfrac{\pi}{12}$	**M1**	**FT** provided at least **B1** awarded
	0.495 or 0.4954[57…] rot to 4 or more sf	**A1**	**dep** on all previous marks awarded
2(b)	$\dfrac{1}{4}\ln(4x-3) - \dfrac{x^{-2}}{2} \; (+c)$ oe, isw or $\dfrac{1}{4}\ln(x-0.75) - \dfrac{x^{-2}}{2} \; (+c)$ oe, isw	**B3**	**B2** for $\dfrac{1}{4}\ln(4x-3)$ or $\dfrac{1}{4}\ln(x-0.75)$ or **B1** for $\dfrac{1}{4}\ln 4x-3$ or $\dfrac{1}{4}\ln x - 0.75$ or $k\ln(4x-3)$ or $k\ln(x-0.75)$ where $k \neq \dfrac{1}{4}$ and **B1** for $\dfrac{x^{-2}}{-2}$ oe

Question	Answer	Marks	Partial Marks
3(a)	$7x^2 + 9x + 5[=0]$ or $-7x^2 - 9x - 5[=0]$	**B2**	**B1** for two terms correct in $7x^2 + 9x + 5 = 0$ or at most one term incorrect in $12x^2 + 11x + 2 = 5x^2 + 2x - 3$ oe
	$9^2 - 4(7)(5)$ or $(-9)^2 - 4(-7)(-5)$ oe	**M1**	**FT** *their* 3-term quadratic
	-59 and no real roots or $81 - 140 < 0$ and no real roots oe	**A1**	
3(b)	$\left(\sqrt[3]{x}\right)^2 + 4\sqrt[3]{x} - 12 = 0$ oe soi or $y = \sqrt[3]{x}$ and $y^2 + 4y - 12 = 0$ oe soi	**B1**	
	Factorises or solves *their* 3-term quadratic in $\sqrt[3]{x}$	**M1**	**FT** *their* 3-term quadratic in $\sqrt[3]{x}$ or a stated substituted unknown
	$x = 8$ $x = -216$	**A2**	**A1** for $\sqrt[3]{x} = 2$ $\sqrt[3]{x} = -6$
4(a)	33	**B1**	
4(b)(i)	$6\left(\dfrac{1}{2}\right)^3 + \left(\dfrac{1}{2}\right)^2 - 12\left(\dfrac{1}{2}\right) + 5 = 0$ oe or $6\left(\dfrac{1}{8}\right) + \dfrac{1}{4} - \dfrac{12}{2} + 5 = 0$ oe or $\dfrac{3}{4} + \dfrac{1}{4} - 6 + 5 = 0$ oe	**B1**	
4(b)(ii)	Finds the quadratic factor $3x^2 + 2x - 5$	**M2**	**M1** for any two terms correct in $3x^2 + 2x - 5$
	$(2x - 1)(3x + 5)(x - 1)$ oe	**A1**	If 0 scored then **SC2** for justifying $x - 1$ as a factor and writing down $(2x - 1)(3x + 5)(x - 1)$ without any incorrect work seen
4(b)(iii)	$[\sin\theta = 0.5]$ $\theta = 30$ nfww $[\sin\theta = 1]$ $\theta = 90$ nfww and no value of θ from $3\sin\theta + 5 = 0$	**B2**	**B1** for $\sin\theta = 0.5$ or $\theta = 30$ or **B1** for $\sin\theta = 1$ nfww or $\theta = 90$ nfww

Question	Answer	Marks	Partial Marks
5	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 10\mathrm{e}^{2x-1}\,[+0]$ oe	M2	**M1** for $\dfrac{\mathrm{d}y}{\mathrm{d}x} = k\mathrm{e}^{2x-1}$, $k \neq 10$ or **SCM1** for $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 10\mathrm{e}^{2x-1} + c$ where c is algebraic or numerical
	$\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=1} = 10\mathrm{e}$ and $y = 6\mathrm{e}$	A1	**FT** *their* $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ provided **M1** or **SCM1** awarded and a value is found
	$y - 6\mathrm{e} = 10\mathrm{e}(x-1)$ oe or $y = 10\mathrm{e}x + c$ and $6\mathrm{e} = 10\mathrm{e} + c$	M1	**FT** *their* $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=1}$ and *their* y
	$y = 10\mathrm{e}x - 4\mathrm{e}$ isw	A1	
	[x-coordinate of $P =$] 0.4 oe, isw	A1	**dep** on correct equation of tangent with exact values
6(a)	Convincing correct statement from which the answer can be easily determined e.g. $\sin^3 x \left(\dfrac{1}{\sin x} \times \dfrac{\sin x}{\cos x} \right)$ oe or $\sin^2 x \left(\sin x \times \dfrac{1}{\sin x} \times \dfrac{1}{\cot x} \right)$ oe or $\sin^3 x \left(\dfrac{1}{\sin x} \div \dfrac{\cos x}{\sin x} \right) = \sin^3 x \left(\dfrac{1}{\cos x} \right)$ oe or $\sin^3 x \left(\dfrac{1}{\sin x} \div \dfrac{1}{\tan x} \right) = \sin^3 x \left(\dfrac{1}{\sin x} \times \tan x \right)$ oe	2	**M1** for either $\mathrm{cosec}\,x$ correctly written as $\dfrac{1}{\sin x}$ oe seen in a correct expression or for $\cot x$ correctly written as $\dfrac{\cos x}{\sin x}$ or $\dfrac{1}{\tan x}$ oe seen in a correct expression e.g. $\sin^3 x \left(\mathrm{cosec}\,x \times \tan x \right)$ or $\sin^3 x \left(\mathrm{cosec}\,x \div \dfrac{\cos x}{\sin x} \right)$ or $\sin^3 x \left(\dfrac{1}{\sin x} \div \dfrac{\cos x}{\sin x} \right)$ or $\sin^3 x \left(\dfrac{1}{\sin x} \div \dfrac{1}{\tan x} \right)$
	Correct completion to given answer: $\sin^2 x \tan x$	A1	

Question	Answer	Marks	Partial Marks
6(b)	Factorises: $\tan x\left(\cos^2 x - \dfrac{1}{2}\right) = 0$ oe or $\tan x\left(\dfrac{1}{2} - \sin^2 x\right) = 0$ oe or correctly rewrites and then factorises: $\sin x\left(\cos^2 x - \dfrac{1}{2}\right) = 0$ oe or $\sin x\left(\dfrac{1}{2} - \sin^2 x\right) = 0$ oe or $\tan x\left(1 - \tan^2 x\right) = 0$ oe	M1	Note: division by $\tan x$ is **M0** Note: division by $\sin x$ is **M0**
	[$\tan x = 0$ or $\sin x = 0$] [$x =$] 0	A1	
	$\cos x = [\pm]\sqrt{\dfrac{1}{2}}$ oe or $\sin x = [\pm]\sqrt{\dfrac{1}{2}}$ oe or $\tan x = [\pm]1$	M1	nfww
	[$x =$] $\pm\dfrac{\pi}{4}$ or ± 0.785 or ± 0.7853 to ± 0.7854, $\pm\dfrac{3\pi}{4}$ or ± 2.36 or ± 2.356 to ± 2.3562	A2	with no extras in range; nfww **A1** for any two out of four correct, ignoring extras
7(a)	282 240	2	**M1** for $9! - 2! \times 8!$ oe
7(b)	120	2	**M1** for $5!$ or $^5P_5$ oe
8(a)	Points plotted at <table><tr><td>x</td><td>15</td><td>30</td><td>45</td><td>60</td><td>75</td></tr><tr><td>lny</td><td>2.3</td><td>2.5 or 2.6</td><td>3.1</td><td>3.5 or 3.6</td><td>3.9</td></tr></table> soi and single, ruled, straight line of best fit drawn	B2	**B1** for at least 4 correctly plotted points

Question	Answer	Marks	Partial Marks
8(b)	$\ln y = 0.03x + 1.8$	2	**M1** for m = awrt 0.02 to awrt 0.03 or c = awrt 1.7 to awrt 2.0 or for the straight line form in terms of $\ln A$ and k: $\ln y = \ln A + kx$[lne]
	$A = 6$ or $A = 7$ and $k = 0.03$ or $k = 0.02$	B3	Must have been found using linear points **or** linear equation **B2** for $A = 6$ or $A = 7$ or A in range: awrt 6 or awrt 7 or **B1 FT** for $\ln A = their$ 1.8 or $A = \mathrm{e}^{their1.8}$ and **B1 FT** for $k = 0.03$ or 0.02 or k in range: awrt 0.02 or awrt 0.03 **Maximum of 2 marks if one or both values not rounded to 1 sf** If **B0** scored, award **SC1** for $A = 6$ or $A = 7$ and **SC1** for $k = 0.03$ or $k = 0.02$ found not using transformed data

Question	Answer	Marks	Partial Marks
8(c)	A value of x in range $33 \leqslant x \leqslant 37.5$ nfww, isw	**2**	**M1** for $\ln y = 2.8$ or $2.83[32...]$ OR A value of x in range $29.5 \leqslant x < 33$ or $37.5 < x \leqslant 45$ nfww OR **M1 STRICT FT** for $x = \dfrac{\ln 17 - their \ln A}{their\ k}$ **STRICT FT** *their* stated ln$A$ and *their* stated k or *their* stated linear equation in **(b)** OR $x = \dfrac{1}{their\ k} \ln\left(\dfrac{17}{their A}\right)$ **STRICT FT** *their* stated A and *their* stated k or *their* stated exponential equation in **(b)**

Question	Answer	Marks	Partial Marks
9	$A(2, 0)$ or $x = 2$ [$x = 6$]	**2**	**M1** for factorising or solving $32x - 4x^2 - 48 = 0$
	Finds equation CD/ x-coordinate of C or D or maximum point : $x = 4$	**2**	**M1** for $\dfrac{2+6}{2}$ or $32 - 8x = 0$
	$D(4, 8)$ or [equation AB is $y =$] $4x - 8$	**2**	**M1** for $y = \dfrac{2}{3} \times 12$ or $m_{AB} = \dfrac{12-0}{5-2}$ soi
	$\left[\dfrac{32}{2}x^2 - \dfrac{4}{3}x^3 - 48x\right]_2^4$ or $\left[\dfrac{28}{2}x^2 - \dfrac{4}{3}x^3 - 40x\right]_2^4$	**B1**	**must be seen**
	Correct plan **including** correct substitution of upper and lower limits at some point e.g. $\left[16x^2 - \dfrac{4}{3}x^3 - 48x\right]_2^{their4} - \dfrac{1}{2} \times (their4 - 2) \times their8$ or $\left[16x^2 - \dfrac{4}{3}x^3 - 48x\right]_2^{their4} - \left[2x^2 - 8x\right]_2^{their4}$ or $\left[14x^2 - \dfrac{4}{3}x^3 - 40x\right]_2^{their4}$	**M1**	**dep** on attempt to integrate **FT** *their* 4 and *their* 8 if needed or **FT** *their* 4 and *their* $4x - 8$ of the form $mx + c$ if needed or **FT** *their* 4 and $(32 - their4)x - 4x^2 + (-48 - their(-8))$ providing clear evidence of the derivation of this has been seen
	$\dfrac{40}{3}$ isw or 13.3[33….] **nfww**	**A1**	**dep on all previous marks awarded**
10(a)	Valid explanation using f: f is one-one oe	**B1**	
10(b)	Complete method to find inverse function: Swaps the variables and rearranges or rearranges and swaps the variables	**M1**	Condone one sign or arithmetic error but must have the correct order of operations
	$\left[f^{-1}(x) =\right] -\sqrt{\ln x - 3}$ isw or $\left[f^{-1}(x) =\right] -\sqrt{\ln \dfrac{x}{e^3}}$ oe isw	**A2**	**A1** for $\left[f^{-1}(x) =\right] [\pm]\sqrt{\ln x - 3}$ or $\left[f^{-1}(x) =\right] [\pm]\sqrt{\ln \dfrac{x}{e^3}}$ oe
	Domain f^{-1}: $x > e^3$	**B1**	
	Range f^{-1}: $f^{-1} < 0$	**B1**	

Question	Answer	Marks	Partial Marks
10(c)	$g(x) = f^{-1}(e^{2x})$ soi or $g(x) = -\sqrt{\ln e^{2x} - 3}$	M1	**FT** *their* expression for f^{-1}
	$-\sqrt{2x-3}$	A1	If 0 scored, allow **SCB1** for $-\sqrt{2x-3}$ found from solving $(g(x))^2 + 3 = 2x$ and using existence of composite functions to deduce that the square root must be negative